Thorsten Theobald:
Real Algebraic Geometry and Optimization
AMS Graduate studies in Mathematics, volume 241

Thanks to the readers for helping to improve the book by sending corrections and comments.
Version of: October 16, 2025.

Erratum

Chapter 1

P. 12, l. 11 from bottom: should say $0 \leq t \leq 2n-2$
P. 18, Exer. 5: insert 'every root α satisfies'
P. 18, Exer. 10: should say '$p = x^3 + \cdots$'

Chapter 2

P. 30, Exer. 2.3: should say '$P = \{(x,y) \in \mathbb{R}^2 \ldots$'.

Chapter 3

P. 35, l. 11 from bottom: Lemma 3.4 should be Theorem 3.4
P. 41, l. 2 from bottom: should say 'read off from'.
P. 42, l. 4: should say 'matrix S_2'.
P. 44, l. 10 of Ex. 3.20: should say '$-\frac{1}{4}a^2 + b$'
P. 44, l. 1 of Thm. 3.27: should say 'real closed field'
P. 48, last line of proof: should say '$\exists x \Phi$'

Chapter 4

P. 59, l. 4 from bottom: should say 'deg$(sf + rg)$'
P. 61, l. 10: should say 'polynomials f_i'
P. 61, l. 7: should say 'through a double'

Chapter 6

P. 107, l. 1: should say 'resultant of p'

P. 117, l. 5 and l. 9: '$\subset \Sigma[x,y]$' and '$\subset \sigma[x]$' should say '$\subset \mathbb{R}[x,y]$' and '$\subset \mathbb{R}[x]$', respectively.

P. 117, last three lines of proof of Thm. 6.28: should say 'and H' is a product of even powers of $h_1,\ldots,h_t$. This proves the claim.'

P. 122, proof of Ex. 6.38, l. 2 from bottom: after '$1-x$,' insert '$(1+x)^2$, $(1-x)^2$'

P. 123, proof of Thm. 6.39: replace '..., due to the identity $N_1N_2\ldots$' by the following: ..., which follows from setting $g := p \pm q$, $h := p \mp q$ and considering the identity

$$\begin{aligned} \frac{N_1^2}{2} \mp pq &= \frac{1}{4}\Big(N_1^2 + h^2 + \frac{1}{2N_1}\left((N_1+g)(N_1^2-g^2) + (N_1-g)(N_1^2-g^2)\right)\Big) \\ &= \frac{1}{4}\left(N_1^2 + h^2 + \frac{1}{2N_1}\left((N_1+g)^2(N_1-g) + (N_1-g)^2(N_1+g)\right)\right). \end{aligned}$$

P. 126, l. 8: should say 'the function $\sqrt{\bar{\sigma}_{m+1}}$'

P. 126, last paragraph: There is a problem in the proof, since the assertion on p. 127 that $h_1,\ldots,h_{n+1}$ have the Handelman property does not work as described. To fix the proof, replace the paragraph by the following: 'Let p be a strictly positive polynomial on the unit ball $K = \{x \in \mathbb{R}^n \,:\, g_1(x) \geq 0\}$. Further let P be a polytope with facets as tangent hyperplanes of K, such that P approximates K sufficiently well and p is still positive on P. We write P as $P = \{x \in \mathbb{R}^n \,:\, h_i(x) \geq 0,\ 1 \leq i \leq m\}$ with affine-linear polynomials h_i. Then there exists a Handelman representation

$$p = \sum_{\beta} c_\beta h_1^{\beta_1} \cdots h_m^{\beta_m}$$

with nonnegative coefficients c_β.' As before, this gives a representation $p = \sigma_0(1-\sum_{i=1}^n x_i^2)+\sigma_1$ with sums of squares σ_0 and σ_1. Conclude with 'This shows $p \in \mathrm{QM}(g_1)$ and thus the single polynomial $1-\sum_{i=1}^n x_i^2$ has the Putinar property.'

P. 127, l. 1 of proof: should say 'Since $\mathrm{QM}(g_1,\ldots,g_m)$ is Archimedean'

P. 131, l. 1 and 2: should say $\gamma' = \gamma(1+\tau/2)^2$

P. 131, l. 8 from bottom: should say $\sum_{e\in\{0,1\}^r} \sigma_e h_1^{e_1}\cdots h_r^{e_r}$

P. 131, l. 7 from bottom: should say $N \pm h \in \mathrm{QM}(g_1,\ldots,g_m)$

Chapter 7

P. 144, l. 3 from bottom: should say $\frac{-729}{4096} \approx -0.17798$

P. 148, l. 6 or proof: should say $\sum_{i=1}^{k} \mathcal{L}_i(q_i, q_i)$

P. 151, l. 2: should say 'moment relaxation.'

P. 151, l. 4: should say 'with rank $M_d(y^*)$

P. 151, l. 5: should say 'of the form $M_d(y^*) = v^*(v^*)^T$'

P. 153, l. 2: should say $z = (z_0, z_1, \ldots, z_d)$

P. 153, l. 4 of proof of Thm. 7.19: should say $\sum\limits_{i=0}^{d} z_i \sum\limits_{j+k=i} q_j q_k$

P. 154, l. 4 of paragraph after proof: should say $\mathcal{M}_n \subset \mathcal{P}[x]$

P. 155, l. 2: should say $t \in \mathbb{N}$

P. 160, l. 6 of equation (7.21): should say $\mathcal{L}_{g_j, \leq t - \lceil \deg(g_j)/2 \rceil}$

P. 164, l. 3 after Ex. 7.36: should say $\mathcal{V}_{\mathbb{C}}(I)$

P. 164, l. 6 from bottom: should say 'square root of $p(a)$'

P. 164, l. 3 from bottom: should say $q(x) = 0$ for all $x \in \mathcal{V}_{\mathbb{C}}(I)$

P. 165, l. 4: should say $r_0(a) = p(a)$

P. 166, l. 3 and 4: should say $\mathcal{L}_{g_j, \leq t - \lceil \deg(g_j)/2 \rceil}$ and $L \in (\mathbb{R}[x]_{\leq 2t})^*$, respectively

P. 168, last line of proof: should say $|\mathcal{B}| = \dim(\mathbb{R}[x]/I)$

P. 168, l. 2 after proof: should say $\lambda_1, \ldots, \lambda_r > 0$

P. 169, last two lines of proof of Cor. 7.42: should say 'evaluating $\mathcal{L}$ at the polynomials p_k'

P. 170, l. 1 of Exer 7.4: should say 'for every $g_1, \ldots, g_m \in \mathbb{R}[x]$ affine linear with'

P. 170, l. 3 of Exer 7.4: should say 'Theorem 7.5'

Chapter 10

P. 239, l. 4 of proof: should say $z_2 \geq z_3 \exp\left(\frac{z_1}{z_3}\right)$

P. 239, l. 6 of proof: should say $\geq s_1 z_1 + (-s_1) \exp\left(\frac{s_3}{s_1} - 1\right) z_3 \exp\left(\frac{z_1}{z_3}\right) + s_3 z_3$

P. 239, last line of proof: should say '$z \in K_{exp}$.'

P. 244, l. 1 from bottom: should say 'equal to $-d$.'

P. 245, l. 3 before Thm. 10.9 should say $\sum_{\alpha \in \mathcal{A}} \nu_\alpha = 1$