Correction to:

Trading degree for dimension in the section conjecture:

The non-abelian Shapiro Lemma

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I am grateful to Florian Herzig who noticed and communicated to me the following bugs in Proposition 8 of [Sti10], including appropriate corrections.

- The proof of the surjectivity part contains an inaccuracy. The formula for $b_{s,t}$ contains values a_y for $y \in Y$, which are not defined. These are set to 1 which simplifies the formula for $b_{s,t}$.
- Similarly, the proof of the injectivity part contains an inaccuracy. The definition of the function $f(s)$ must be changed.

Below, we give an approriately modified and expanded proof. For notation we refer to [Sti10].

Proposition 8. *The Shapiro map* $\mathrm{sh}^1 : \mathrm{H}^1(G, \mathrm{ind}_H^G(N)) \to \mathrm{H}^1(H, N)$ *is bijective.*

New proof. A 1-cocycle $s \mapsto b_s$ for G with values in $\mathrm{ind}_H^G(N)$ is given by $b_{s,t} = b_s(t) \in N$ for all $s, t \in G$ such that

(i) $b_{s,ht} = \vartheta(h)(b_{s,t})$ for all $s, t \in G$ and $h \in H$, and

(ii) $b_{st,g} = b_{s,g} b_{t,gs}$ for all $s, t, g \in G$.

The map sh^1 on the level of cocycles maps b to $h \mapsto b_{h,1}$.

Surjectivity. We choose a set of representatives $Y \subset G$ for $H\backslash G$ with $1 \in Y$ and obtain maps $\gamma : G \to H$ and $y : G \to Y$ such that $g = \gamma_g y_g$ for all $g \in G$. In particular, $\gamma|_H$ is the identity. Let $a : H \to N$ be a 1-cocycle, in particular $a_1 = 1$. We set

$$b_{s,t} := (a_{\gamma_t})^{-1} a_{\gamma_{ts}}$$

and the following routine calculation shows that b is a 1-cochain with values in $\mathrm{ind}_H^G(N)$

$$\begin{aligned} b_{s,ht} &= (a_{\gamma_{ht}})^{-1} a_{\gamma_{hts}} \\ &= (a_{h\gamma_t})^{-1} a_{h\gamma_{ts}} \\ &= \big(a_h \vartheta(h)(a_{\gamma_t})\big)^{-1} \big(a_h \vartheta(h)(a_{\gamma_{ts}})\big) \\ &= \vartheta(h)\big((a_{\gamma_t})^{-1} a_{\gamma_{ts}}\big) = \vartheta(h)(b_{s,t}) \end{aligned}$$

and a 1-cocyle

$$\begin{aligned} b_{s,g} b_{t,gs} &= (a_{\gamma_g})^{-1} a_{\gamma_{gs}} (a_{\gamma_{gs}})^{-1} a_{\gamma_{gst}} \\ &= (a_{\gamma_g})^{-1} a_{\gamma_{gst}} = b_{st,g} \end{aligned}$$

maping to a

$$\mathrm{sh}^1(b) = \Big(h \mapsto b_{h,1} = (a_{\gamma_1})^{-1} a_{\gamma_h} = (a_1)^{-1} a_h = a_h\Big) = a.$$

Injectivity. Let b, b' be cocycles with $\mathrm{sh}^1(b) \sim \mathrm{sh}^1(b')$ which means that there is a $c \in N$ such that for all $s \in H$

$$b'_{s,1} = c b_{s,1} (\vartheta(s)(c))^{-1}.$$

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We define $f \in \mathrm{ind}_H^G(N)$ by

$$f(s) := (b'_{s,1})^{-1} c b_{s,1}$$

for all $s \in G$. Indeed, for $h \in H$ and $s \in G$ we compute using the cocycle equation

$$\begin{aligned} f(hs) &= \big(b'_{hs,1}\big)^{-1} c b_{hs,1} \\ &= \big(b'_{h,1} b'_{s,h}\big)^{-1} c \big(b_{h,1} b_{s,h}\big) \\ &= \big(b'_{s,h}\big)^{-1} \big((b'_{h,1})^{-1} c b_{h,1}\big) \big(b_{s,h}\big) \\ &= \big(b'_{s,h}\big)^{-1} \big(\vartheta(h)(c)\big) \big(b_{s,h}\big) \\ &= \vartheta(h)\big((b'_{s,1})^{-1} c b_{s,1}\big) = \vartheta(h)(f(s)) \end{aligned}$$

as claimed. For all $s, t \in G$, a routine calculation based on the cocycle equation yields

$$\begin{aligned} f(t) b_{s,t} f(ts)^{-1} &= (b'_{t,1})^{-1} c b_{t,1} b_{s,t} \big((b'_{ts,1})^{-1} c b_{ts,1}\big)^{-1} \\ &= (b'_{t,1})^{-1} c b_{ts,1} \big((b'_{ts,1})^{-1} c b_{ts,1}\big)^{-1} \\ &= (b'_{t,1})^{-1} b'_{ts,1} = b'_{s,t} \end{aligned}$$

which translates into $b \sim b'$. □

References

[Sti10] Stix, J., Trading degree for dimension in the section conjecture: The non-abelian Shapiro Lemma, *Mathematical Journal of Okayama University* **52** (2010), 29–43.

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