GOETHE UNIVERSITÄT FRANKFURT AM MAIN

WiSe 2019/20
Prof. Dr. Raman Sanyal
Sebastian Manecke

Bewertungen

http://tinygu.de/VL-Bew19

3. Übungsblatt — Besprechung am 25. November 2019

Exercise 1. For $\varnothing \neq S \subseteq \mathbb{R}^d$ let the following four maps $\phi_* : \mathcal{P}^d \to \mathbb{Z}$ be given for every non-empty polytope P as

- $\phi_{\subseteq S}(P) = 1$ if and only if $P \subseteq S$;
- $\phi_{\supseteq S}(P) = 1$ if and only if $P \supseteq S$;
- $\phi_{\cap S}(P) = 1$ if and only if $P \cap S \neq \varnothing$;
- $\phi_{\cup S}(P) = 1$ if and only if $P \cup S$ is convex,

as well as $\phi_*(\varnothing) = 0$.

i) For a fixed non-empty polytope S, which of these functions is a valuation?

ii) Let $H \subset \mathbb{R}^d$ be a hyperplane and let $H^<$ be the *open* halfspace induced by H. Show that $\phi_{\subseteq H^<}$ is a valuation. Which polytopes P have $\phi_{\subseteq H^<}(P) = 1$?

Exercise 2. The Minkowski-sum $A + B$ of two sets $A, B \subseteq \mathbb{R}^d$ is defined as

$$A + B \;:=\; \{a + b : a \in A, b \in B\}\,.$$

Let $P, P', Q \in \mathcal{P}_d$ be polytopes.

i) Show that $(P \cup P') + Q = (P + Q) \cup (P' + Q)$

ii) Assume $P \cup P'$ is convex. Show that $(P \cap P') + Q = (P + Q) \cap (P' + Q)$

iii) Let $\phi : \mathcal{P}^d \to G$ be a valuation. Show that for $Q \in \mathcal{P}^d$ the map $\phi_Q : \mathcal{P}^d \to G$, $\phi_Q(P) := \phi(P + Q)$ is a valuation.

Exercise 3. If $\phi : \{\text{relative open polyhedra}\} \to (G, +)$ is any map such that for all relative open polyhedron Q° and all hyperplanes H one has

$$\phi(Q^\circ) \;=\; \phi(Q^\circ \setminus H^+) + \phi(Q^\circ \setminus H^-) + \phi(Q^\circ \cap H)\,,$$

then ϕ can be extended to a valuation on $B\mathcal{Q}^d$.