MfN I Blatt 7 Musterlösung

Nr. 1

mögliche Unterteilung und Zwischenstellen:

Teilintervall	Zwischenstelle z_i
0 bis 0.15	0.1
0.15 bis 0.3	0.2
0.3 bis 0.5	0.4
0.5 bis 0.65	0.6
0.65 bis 0.85	0.75
0.85 bis 1	0.9

x_{i-1} x_i

Ansatz: $I_z = \sum_{i=1}^{n} f(z_i) \cdot (x_i - x_{i-1})$

$= e^{0.1^2} \cdot (0.15-0) + e^{0.2^2} \cdot (0.3-0.15) + e^{0.4^2} \cdot (0.5-0.3)$

$+ e^{0.6^2} \cdot (0.65-0.5) + e^{0.75^2} \cdot (0.85-0.65) + e^{0.9^2} \cdot (1-0.85)$

$\approx \underline{\underline{1.44553}}$

Nr. 2

a) $\int_0^{2\pi} \cos(x)\,dx = [\sin(x)]_0^{2\pi} = \sin(2\pi) - \sin(0) = \underline{\underline{0}}$

b) $\int_0^1 \sqrt[3]{x}\,dx = \left[\frac{3}{4} \sqrt[3]{x^4}\right]_0^1 = \left(\frac{3}{4} \cdot \sqrt[3]{1^4}\right) - \left(\frac{3}{4} \cdot \sqrt[3]{0^4}\right) = \frac{3}{4} - 0 = \underline{\underline{\frac{3}{4}}}$

c) $\int_1^2 \frac{x^3+1}{x}\,dx = \int_1^2 \left(x^2 + \frac{1}{x}\right)dx = \left[\frac{1}{3}x^3 + \ln(x)\right]_1^2 = \left(\frac{1}{3} \cdot 2^3 + \ln(2)\right) - \left(\frac{1}{3} \cdot 1^3 + \underbrace{\ln(1)}_{0}\right) = \frac{8}{3} - \frac{1}{3} + \ln(2) = \underline{\underline{\frac{7}{3} + \ln(2)}}$

d) $\int_0^{\frac{\pi}{4}} \tan(x)\,dx = -\int_0^{\frac{\pi}{4}} \frac{\overbrace{-\sin(x)}^{f'(x)}}{\underbrace{\cos(x)}_{f(x)}}\,dx = -\left[\ln|(\cos(x))|\right]_0^{\frac{\pi}{4}} = -\left(\left(\ln\left|\left(\cos\left(\frac{\pi}{4}\right)\right)\right|\right) - \left(\ln|(\cos(0))|\right)\right)$

$= -\ln\left(\frac{\sqrt{2}}{2}\right) + \ln(1) = \underline{\underline{-\ln\left(\frac{\sqrt{2}}{2}\right)}}$

Nr. 3

a) $\int_0^1 \overbrace{x^2}^{g(x)} \cdot \overbrace{e^x}^{f'(x)}\,dx = \left[e^x \cdot x^2\right]_0^1 - \int_0^1 2x \cdot e^x\,dx = \left[e^x \cdot x^2\right]_0^1 - \left[e^x \cdot 2x\right]_0^1 + \int_0^1 2 \cdot e^x\,dx$

$= \left[e^x \cdot x^2\right]_0^1 - \left[e^x \cdot 2x\right]_0^1 + \left[2e^x\right]_0^1 = (e-0) - (2e-0) + (2e-2) = e-2 \approx \underline{\underline{0.7183}}$

b) $\int_1^e \underbrace{x}_{f'(x)} \cdot \underbrace{\ln(x)}_{g(x)}\,dx = \left[\frac{1}{2}x^2 \cdot \ln(x)\right]_1^e - \int_1^e \frac{1}{2}x^2 \cdot \frac{1}{x}\,dx = \left[\frac{1}{2}x^2 \cdot \ln(x)\right]_1^e - \int_1^e \frac{1}{2}x\,dx = \left[\frac{1}{2}x^2 \cdot \ln(x)\right]_1^e - \left[\frac{1}{4}x^2\right]_1^e$

$= \left(\frac{1}{2}e^2 \cdot \ln(e)\right) - \left(\frac{1}{2} \cdot 1^2 \cdot \ln(1)\right) - \left(\frac{1}{4} \cdot e^2\right) + \left(\frac{1}{4} \cdot 1^2\right)$

$= \frac{1}{2}e^2 - \frac{1}{4}e^2 + \frac{1}{4} = \frac{1}{4}e^2 + \frac{1}{4} \approx \underline{\underline{2.0972}}$

c) $\int_0^{\frac{\pi}{2}} \underbrace{\sin(x)}_{g(x)} \cdot \underbrace{\cos(x)}_{f'(x)}\,dx = \left[\sin(x) \cdot \sin(x)\right]_0^{\frac{\pi}{2}} - \int_0^{\frac{\pi}{2}} \cos(x) \cdot \sin(x)\,dx$

$2\int_0^{\frac{\pi}{2}} \cos(x) \cdot \sin(x)\,dx = \left[\sin^2(x)\right]_0^{\frac{\pi}{2}} = \sin^2\left(\frac{\pi}{2}\right) - \sin^2(0) = 1 \quad |:2$

$\int_0^{\frac{\pi}{2}} \cos(x) \cdot \sin(x)\,dx = \underline{\underline{\frac{1}{2}}}$

d) $\int_0^{2\pi} \underbrace{e^x}_{f'(x)} \cdot \underbrace{\cos(x)}_{g(x)}\,dx = \left[e^x \cdot \cos(x)\right]_0^{2\pi} - \int_0^{2\pi} e^x \cdot (-\sin(x))\,dx = \left[e^x \cdot \cos(x)\right]_0^{2\pi} + \int_0^{2\pi} \underbrace{e^x}_{f'(x)} \cdot \underbrace{\sin(x)}_{g(x)}\,dx$

$= \left[e^x \cdot \cos(x)\right]_0^{2\pi} + \left[e^x \cdot \sin(x)\right]_0^{2\pi} - \int_0^{2\pi} e^x \cdot \cos(x)\,dx$

$2\int_0^{2\pi} e^x \cdot \cos(x) = \left[e^x \cdot \cos(x)\right]_0^{2\pi} + \left[e^x \cdot \sin(x)\right]_0^{2\pi}$

$= e^{2\pi} - 1 \quad \Leftrightarrow \int_0^{2\pi} e^x \cdot \cos(x)\,dx = \frac{e^{2\pi}-1}{2} \approx \underline{\underline{267.246}}$