Luis G. Gorostiza, CINVESTAV, Mexico

Title:

Oscillatory fractional Brownian motion and related processes arising from particle systems

Abstract:

Oscillatory fractional Brownian motion with parameter $H \in (1/2, 1)$ is a centered Gaussian process $\xi = (\xi_t)_{t \ge 0}$ with covariance function

$$Cov(\xi_s,\xi_t) = \frac{1}{2} \left(s^{2H} h_s + t^{2H} h_t - |s - t|^{2H} h_{|s-t|} \right), \quad s,t \ge 0,$$

where

$$h_t = \sum_{j=-\infty}^{\infty} (ba^j t)^{-2H} (e^{-ba^j t} - 1 + ba^j t), \quad t > 0, \quad h_0 = 0,$$

constants b > 0 and $a \in (0, 1)$. The function h is periodic in logarithmic scale (i.e., $h_t = h_{at}, t > 0$), and oscillates between two positive values, slower as $t \to \infty$ and faster as $t \to 0$. This process is an oscillatory analogue of fractional Brownian motion with Hurst parameter H (which corresponds to $h \equiv 1$). A related process is an oscillatory analogue of sub-fractional Brownian motion.

In the talk we will show how these processes arise in occupation time fluctuation limits of systems of random walks on a hierarchical group. The oscillations are caused by the discrete and ultrametric structure of the group. Properties of these processes and comparisons with those of their non-oscillatory counterparts will be discussed.

(This is joint work with T. Bojdecki and A. Talarczyk, University of Warsaw).

Reference

T. Bojdecki, L.G. Gorostiza, A. Talarczyk, Oscillatory fractional Brownian motion and hierarchical random walks, arXiv: PR 1201.5084 (2012).