Erased–Word and Erased–Tree Processes: Simplices and Filtrations

Abstract. A *Homogeneously Labelled Bratteli Diagram* (HLBD) is a triple (F, A, Φ), s.t. $F = (F_n)_{n\geq 1}$ and $A = (A_n)_{n\geq 1}$ are sequences of non-empty finite sets and $\Phi = (\Phi_n)_{n\geq 1}$ is a sequence of maps $\Phi_n : F_{n+1} \times A_n \to F_n$. A stochastic process $(X_n, \eta_n)_{n\geq 1}$ is called *central*, if $(X_n, \eta_n) \in F_n \times A_n$, if $X_n = \Phi_n(X_{n+1}, \eta_n)$ almost surely and if η_n is uniform on A_n and independent of $(X_{n+1}, \eta_{n+1}, X_{n+2}, \eta_{n+2}, \ldots)$ for each n. The set of (laws of) central processes always forms a non-empty *metrizable Choquet simplex* and the representation of central processes is closely linked to the backward filtration they generate, which always are *poly-adic backward filtrations*. Two special HLBDs are considered:

1. Words over some finite alphabet Σ. Here $F_n = \Sigma^n$ and $A_n = [n+1] := \{1, 2, \ldots, n+1\}$. For a word $w = (w_1, \ldots, w_{n+1}) \in \Sigma^{n+1}$ and $j \in [n+1]$ one defines $\Phi_n(w, j) \in \Sigma^n$ to be w but with the j-th letter erased.

2. Schröder trees. F_n is the finite set of Schröder trees with n leaves. For $T \in F_{n+1}$ and $j \in A_n = [n+1]$ one defines $\Phi_n(T, j) \in F_n$ to be the Schröder tree obtained by deleting the j-th leaf in T in an appropriate sense, where leaves are enumerated in lexicographic order.

There are close connections to Doob–Martin boundary theory and to exchangeability in random combinatorial structures.

Related Literature.

[1] Choi, Evans. "Doob-Martin compactification of a Markov chain for growing random words sequentially". *Stoch. Processes and their Apps.*, 2017.

[2] Evans, Grübel and Wakolbinger. "Doob-Martin boundary of Rèmys tree growth chain". *The Annals of Probability*, 2017.

[3] Forman, Haulk and Pitman. "A representation of exchangeable hierarchies by sampling from random real trees". *Prob. Theory and related Fields*, 2017.

[4] Gerstenberg. "General Erased–Word Processes: Product-Type Filtrations, Ergodic Laws and Martin Boundaries". *arXiv*, 2017.

[5] Gerstenberg. "Exchangeable interval hypergraphs and limits of ordered discrete structures". *arXiv*, 2018.

[6] Laurent. "Filtrations of the Erased-Word Processes". *Sèminaire de Probabilits XLVIII*, 2016.