Combining frequency-difference EIT with ultrasound modulated EIT (rough sketch of a new idea)

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Introduction

Forward operator of EIT:

$$\text{"conductivity"} \mapsto \text{"measurements"}$$

Main principal difficulties in inverting the forward operator:

- inverse problem is highly non-linear
- inverse problem is highly ill-posed
- typical applications involve large modelling errors (electrode positions, boundary geometry, ...)

Introduction

Approaches to alleviate principal difficulties:

- Large modelling errors
  $\rightsquigarrow$ Use difference measurements
- High non-linearity
  $\rightsquigarrow$ Only detect outer shape of inclusions / conductivity changes
- Ill-posedness
  $\rightsquigarrow$ Measure additional interior data (hybrid tomography).

In this talk: combine all of the above.

Mathematical setting

Setting (continuous measurements):

- Apply current g to boundary of a body Ω

$\rightsquigarrow$ Electric potential u that solves

$$\nabla \cdot \gamma_\omega \nabla u = 0, \qquad \gamma_\omega \partial_\nu u|_{\partial\Omega} = g$$

(γ_ω: complex conductivity at frequency Ω)

- Measure boundary voltage $u|_{\partial\Omega}$ for all possible currents g

$\rightsquigarrow$ Current-to-voltage map $\Lambda(\gamma_\omega): \quad g \mapsto u|_{\partial\Omega}$.

$$\gamma_\omega \in L^\infty_+(\Omega) + \mathrm{i}L^\infty(\Omega) \quad \Longrightarrow \quad \Lambda(\gamma_\omega) \in \mathcal{L}(L^2_\diamond(\partial\Omega), L^2_\diamond(\partial\Omega))$$

Inclusion detection with fdEIT

- $\overline{D} \subset \Omega$, D (the "inclusion")
- Conductivity at zero and non-zero frequency:

$$\gamma_0 = \gamma_0^\Omega + \gamma_0^D \chi_D, \quad \gamma_\omega = \gamma_\omega^\Omega + \gamma_\omega^D \chi_D$$

- Roughly:

$$\left\langle g, \left(\Lambda(\gamma_0) - \frac{\gamma_\omega^\Omega}{\gamma_0^\Omega} \Lambda(\gamma_\omega) \right) g \right\rangle \approx \int_D \left(\frac{\gamma_0^\Omega}{\gamma_\omega^\Omega} \gamma_\omega^D - \gamma_0^D \right) |\nabla u_0|^2.$$

fdEIT: Reconstruct D from $\Lambda(\gamma_0) - \frac{\gamma_\omega^\Omega}{\gamma_0^\Omega} \Lambda(\gamma_\omega)$.

US modulated EIT

- Conductivity at zero frequency: γ_0
- Change conductivity in small area B: $\gamma_0(1+\alpha\chi_B)$, $\alpha > 0$ small
- Roughly:

$$\frac{1}{\alpha}\langle g, (\Lambda(\gamma_0) - \Lambda(\gamma_0(1+\alpha\chi_B))\, g\rangle \approx \int_B \gamma_0(x)|\nabla u_0(x)|^2 \, \mathrm{d}x,$$

US-modulated EIT:
$\Lambda(\gamma_0) - \Lambda(\gamma_0(1+\alpha\chi_B)$ yields interior energy $\gamma_0(x)|\nabla u_0(x)|^2$.

Combination

Combining above rough estimates suggests:

Conjecture (rough)

If $B \subset D$, $\alpha > 0$ small enough

$$B \subseteq D \quad \Longrightarrow \quad \Lambda(\gamma_0(1+\alpha\chi_B)) \geq \frac{\gamma_\omega^\Omega}{\gamma_0^\Omega}\Lambda(\gamma_\omega).$$

Idea

Compare measurements at a different frequency with ultrasound-modulated measurements to find inclusions.

Combination

Theorem

Let $\Re\left(\frac{\gamma_0^\Omega}{\gamma_\omega^\Omega}\gamma_\omega^D\right) > 0$,

$D \subseteq \Omega$ open, $\overline{D} \subseteq \Omega$ has connected complement. $B \subseteq \Omega$ ball.
For suff. small α:

$$B \subseteq D \quad \Longrightarrow \quad \Lambda(\gamma_0(1 + \alpha\chi_B)) \geq \frac{\gamma_\omega^\Omega}{\gamma_0^\Omega}\Lambda(\gamma_\omega).$$

For all $\alpha > 0$:

$$B \not\subseteq D \quad \Longrightarrow \quad \Lambda(\gamma_0(1 + \alpha\chi_B)) \not\geq \frac{\gamma_\omega^\Omega}{\gamma_0^\Omega}\Lambda(\gamma_\omega)$$

Why?

- ratio of background conductivities $\frac{\sigma_\omega}{\sigma_0}$ usually known
  (or estimated by minimizing $\Lambda(\gamma_0(1+\alpha\chi_B)) - r\Lambda(\gamma_\omega)$)
- $\Lambda(\gamma_0(1+\alpha\chi_B))$ and $\Lambda(\gamma_\omega)$ are measured.

Approach only requires measurements.

No info on geometry or electrode position needed!

Monotony tests are stable. Results extend to point electrode models.