

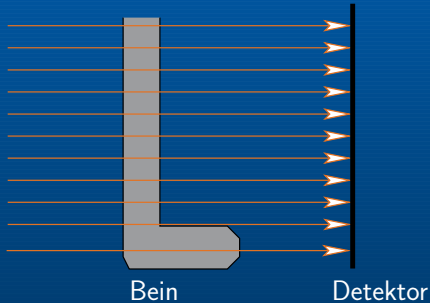


# Computertomographie

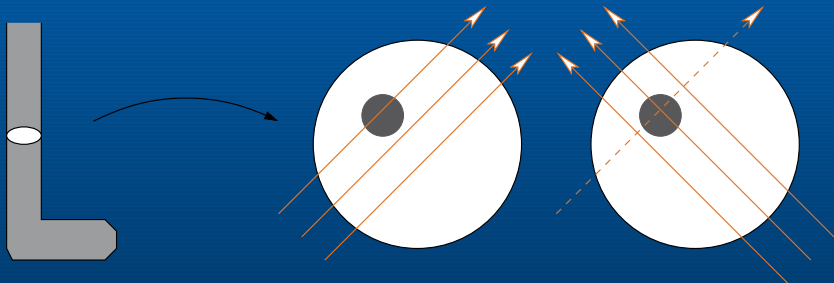
## wie Mathematik Unsichtbares sichtbar macht

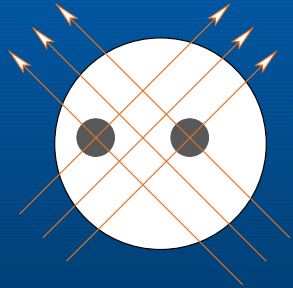
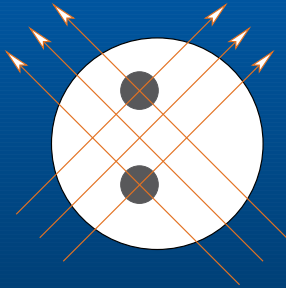
Prof. Dr. Bastian von Harrach

Tag der Mathematik für Schülerinnen und Schüler, 3. Juli 2010.

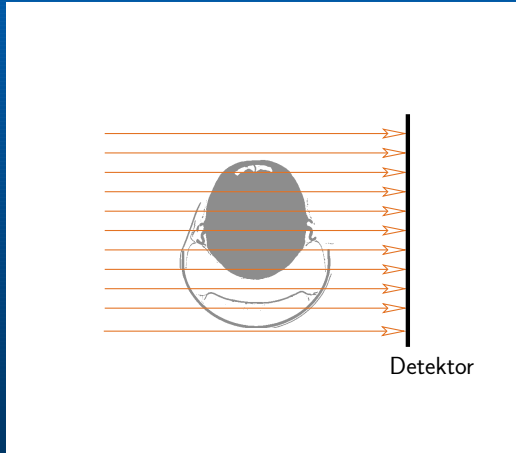


## Röntgen aus zwei Richtungen

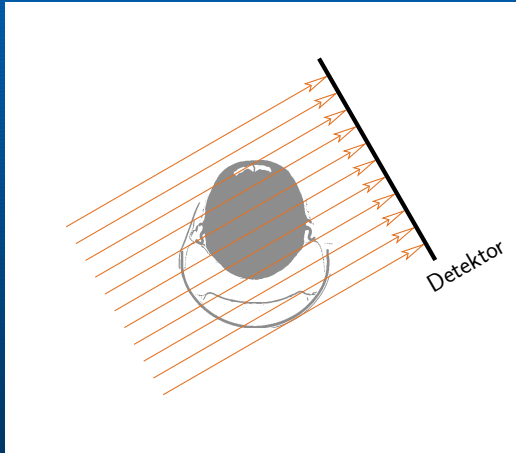




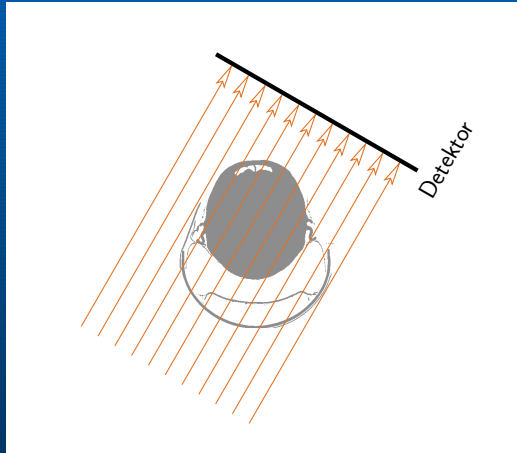
## Röntgen aus 580 Richtungen

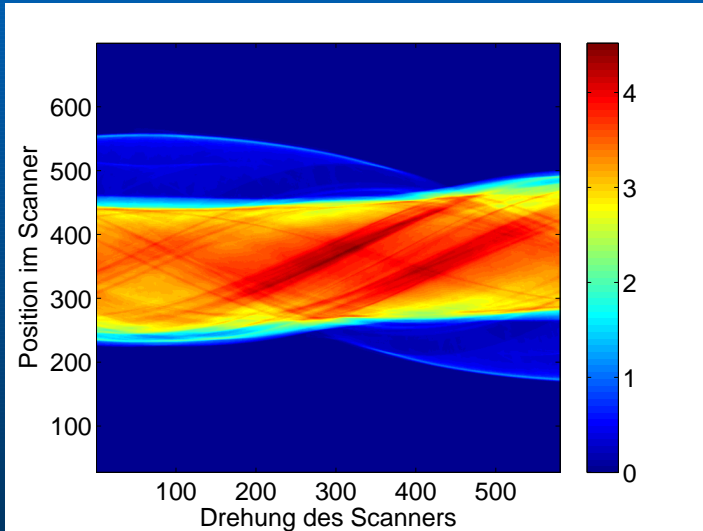


## Röntgen aus 580 Richtungen

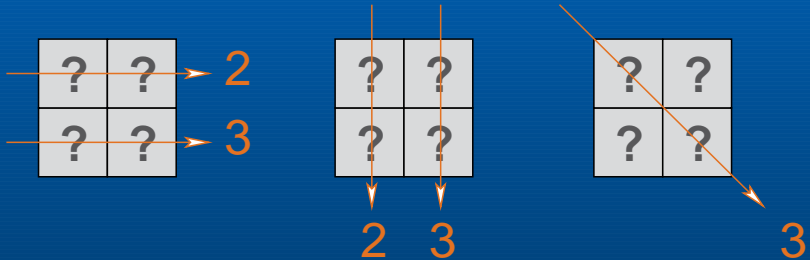


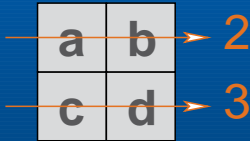
## Röntgen aus 580 Richtungen



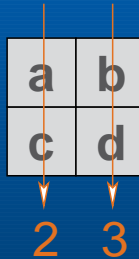




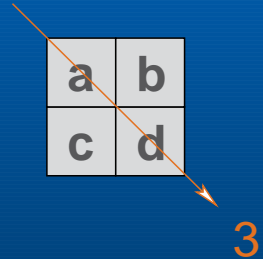




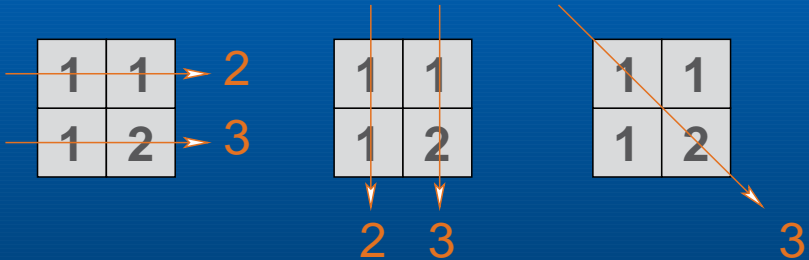
$$\begin{aligned}a + b &= 2 \\ c + d &= 3\end{aligned}$$



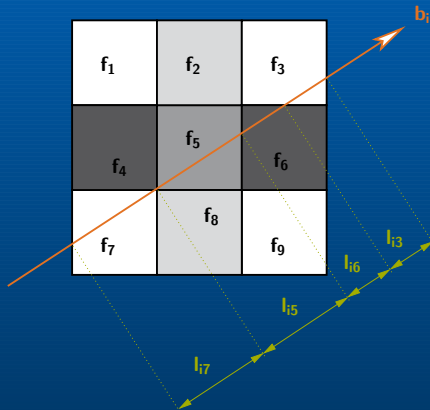
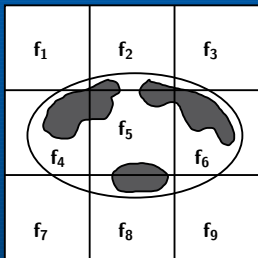
$$\begin{aligned}a + c &= 2 \\ b + d &= 3\end{aligned}$$



$$a + d = 3$$



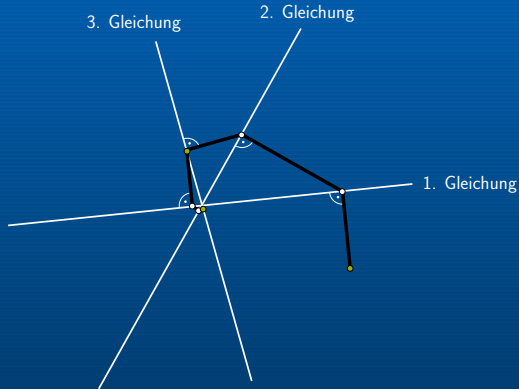
$$\begin{aligned} c + d = 3, b + d = 3, a + d = 3 &\implies a = b = c. \\ a + b = 2 &\implies \underline{a = b = c = 1}. \\ a + d = 3 &\implies \underline{d = 2}. \end{aligned}$$



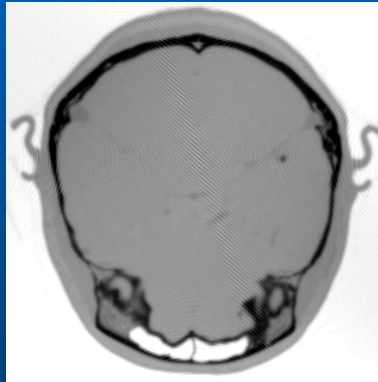
Strahlen  $\rightsquigarrow$  Gleichungen

Pixel  $\rightsquigarrow$  Unbekannte

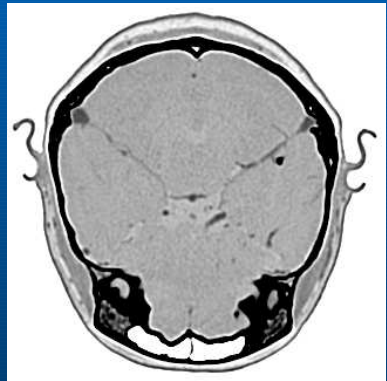
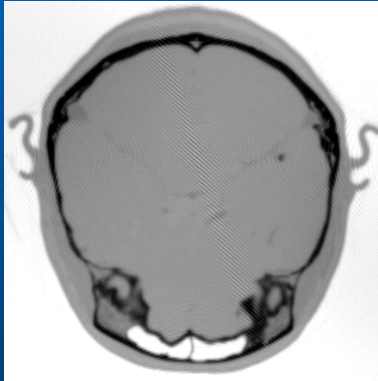
$$l_{i7}f_7 + l_{i5}f_5 + l_{i6}f_6 + l_{i3}f_3 = b_i$$



Kaczmarz-Verfahren für 3 Gleichungen in 2 Unbekannten



Kaczmarz-Rekonstruktion mit  $580 * 672$  Strahlen und  $512 * 512$  Pixeln,  
d.h. ca. 400.000 Gleichungen für ca. 250.000 Unbekannte



Noch bessere Ergebnisse: Inverse Radon-Transformation  
(rechts: Rekonstruktion eines Siemens-Tomographen, ca. 2000)

- ▶ M. Hanke-Bourgeois: *Grundlagen der Numerischen Mathematik und des Wissenschaftlichen Rechnens*, Springer-Verlag, 3. Auflage, 2008.
- ▶ R. Griesmaier: *Computertomographie*, Ausarbeitung zum Tag der offenen Tür, Universität Mainz, 2008.  
[www.math.udel.edu/~griesmai/teaching/comptom.pdf](http://www.math.udel.edu/~griesmai/teaching/comptom.pdf)