An empirical study on mathematical writing in primary education serves as the basis for this article. Students wrote about their problem-solving process, they presented their approaches and negotiated alternative ways of proceeding. Here the focus is put on the emergence of interactional conditions that enable mathematical learning if students discuss alternative approaches based on their written works. Three dimensions are called in to describe how optimized learning conditions emerge within the interaction of the mathematics classroom: the aspect of participation, thematic development and the argumentative aspect. Their interplay provides an enhanced set of terms to approach the aspect of learning in the context of mathematical writing.

INTRODUCTION

“Why should I implement writing in my classes?” (D. Miller 1991, p.518). This question was asked 15 years ago by Diane Miller. In her article she gives an answer similar to those Morgan (1998), Pugalee (2005), Lesser (2000), Gallin and Ruf (Ruf/Gallin 2003) or the NCTM standards (2000) offer today: Writing should be an integral part of the mathematics classroom, because students as well as teachers benefit from this way of working. From a scientific point of view this empirical result is to be appreciated, but at the same time it is unsatisfying (see e. g. Borasi/Rose 1989, p.349). There remains the desire to understand and explain what happens if students write about mathematical concepts and individual problem-solving processes and if they read their work and discuss alternative approaches. How and why does writing contribute to mathematical learning? Where are the positive influences of writing to be situated within the process of learning?

“How does writing improve learning?” (D. Miller 1991, p.516). To me, this is the essential topic to be dealt with. How do situations emerge that make learning possible? What interactional conditions enable mathematical learning? In this article I try to approach these questions. First I am going to give some information concerning the empirical study my work is based on. Afterwards I am going to introduce a set of terms. Finally I am going to present selected empirical results.

THE EMPIRICAL STUDY

In the empirical study, mathematic classes were observed within one course from the first to the third grade. Special emphasis was laid on writing lessons. In order to get hold of the process of writing on the one hand, and to approach aspects of reading,
presenting and discussing on the basis of the written works on the other hand, my empirical study was designed in two phases: the writing phase and the publishing phase. Within the writing process students “externalize” (Bruner 1996) their problem-solving process in a written form. These written works serve as a basis for the subsequent whole-class publishing situation. In this phase several children present their way of proceeding when working on the given task on the board. Alternative approaches are discussed. During the publishing phase all students have their individual works at hand all the time. They might have a quick glance at it at any time.

Most approaches to writing in mathematic classes focus on the products of students’ writing (see e.g. Selter 1994; Ruf/Gallin 2003; Pugalee 2005; Morgan 1998; Borasi/Rose 1989; Fetzer 2003b; Krummheuer/Fetzer 2005). It is assumed that mathematical learning takes place within the writing process. However, doing research on the publishing of the works and the discussion based on the written products is widely neglected. As a consequence my research activities concentrate on these latter aspects (amongst others). In this article whole-class publishing situations are the focus of interest.

If the emphasis is put on interactional situations in the publishing phase it becomes evident that research cannot be restricted to the analysis of the students’ written products. Other aspects gain weight: How do students explain their proceeding? How do they put forward arguments? How do they refer to their own written work and the board? In order to reconstruct how processes of interaction emerge within the publishing phase, 32 mathematics writing classes were videotaped during a three year period. Afterwards transcripts showing verbal and nonverbal aspects as well as the current writings on the board were produced.

Methodologically, processes of interaction are approached by an analysis of interaction. Thus the emerging interactional process can be reconstructed step by step. The interactional analysis is a method derived from conversational analysis (see Eberle 1997; ten Have 1999). In the context of my study I apply the method in the same manner as introduced by Krummheuer and Naujok (1999).

TERMINOLOGICAL BASIS

“How does writing improve learning?” (D. Miller 1991, p.516). How do situations emerge that enable mathematical learning? In order to approach these questions I now introduce a set of terms. In so doing I outline my theoretical framework. In addition I present terms I developed within my empirical study.

To me referring to M. Miller (1986), learning is a matter of participating in interactional processes. Students learn mathematics by being part of and taking part in the ongoing argumentative processes of mathematics classes.
In order to understand interactional processes in class, I refer to three aspects: The participative aspect, thematic development and argumentation. These aspects have been developed empirically (see Fetzer 2006a; Krummheuer/Fetzer 2005). They help to capture interactional processes in the mathematics classroom. Each of these dimensions is explained in the following.

**Participation**

Participation is understood as the students’ or teacher’s participation in classroom interaction. Participating in this educational context can be distinguished as ‘taking part’ on the one hand and ‘being part of’ on the other hand. Taking part is an active form of participating, whereas being part of is a rather receptive one. However, a receptive participant of the classroom interaction may change her/his status of participation and take action (see also Fetzer 2006a).

The following two examples, both taken from a whole-class publishing phase, are meant to explain some terms developed within the empirical study. Just before the first episode begins (transcript 1), Benno has explained on the board how he proceeded in working on the given task. When Benno calls Sonja’s name she asks: “How’d you get the twelve if you (incomprehensible) the two?” Thereupon Benno begins to explain his proceeding again.

<table>
<thead>
<tr>
<th>Person</th>
<th>Aktivität Activity</th>
</tr>
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<tbody>
<tr>
<td>Benno</td>
<td>Sonja-</td>
</tr>
<tr>
<td>Sonja</td>
<td>Wie kummsch n da(nn) auf die zwölf</td>
</tr>
<tr>
<td>&lt;Sonja</td>
<td>wenn du die zwei (unverständlich)-</td>
</tr>
<tr>
<td>&lt;Benno</td>
<td>Von elf Zentimeter</td>
</tr>
<tr>
<td></td>
<td>zeigt erst auf die 11, dann auf die cm</td>
</tr>
<tr>
<td></td>
<td>5cm+6cm=11cm</td>
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<tr>
<td></td>
<td>8mm+4mm=12mm</td>
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<tr>
<td></td>
<td>11cm+12mm=12cm2mm</td>
</tr>
<tr>
<td></td>
<td>habisch-</td>
</tr>
<tr>
<td></td>
<td>zeigt auf die 12 (mm)</td>
</tr>
<tr>
<td></td>
<td>5cm+6cm=11cm</td>
</tr>
<tr>
<td></td>
<td>8mm+4mm=12mm</td>
</tr>
<tr>
<td></td>
<td>11cm+12mm=12cm2mm</td>
</tr>
<tr>
<td></td>
<td>also elf Zentimeter hab ich schon-</td>
</tr>
</tbody>
</table>

**Transcript 1: How’d you get the twelve (See transcription rules, below)**

Regarding the organization of “turn-taking” (Sacks 1996), how does it happen that Sonja takes the role of the current speaker? Benno calls Sonja by name; he selects her as next current speaker. Sonja accepts the turn and starts speaking. This way of taking a turn after being addressed personally I call “accepting a turn”.

In the following example (transcript 2) the second way of turn-taking I could identify within my studies is introduced. The episode occurred a couple of minutes earlier. Sonja is the one who presented her approach to the task on the board. Sabina expresses her embarrassment: “Somehow I don’t get it.” Sonja starts explaining...
Benno did not accept a turn as Sonja did in the first example. He was neither addressed nor was his name called. However, he seems to grab the next speaker’s role. He simply takes hold of it without being called on. If the current speaker does not select the next speaker, if there is no change in turn-taking intended by him or her, but someone gets hold of the turn nevertheless, I call this way of turn-taking “seizing a turn”.

The third way of taking a turn is to “pick it up”. Time and again the current speaker leaves an opening and thus offers the next speaker’s role to somebody else, but does not address anybody specifically. He or she ‘invites’ other members of the interaction to take over. There is the possibility to “refuse” an offered turn. (See also Fetzer 2006b).

**Thematic development**

In introducing terms concerning thematic development I want to describe the specific interactional situation of the publishing phase. During the writing process the students produced written works. While the publishing phase proceeds, each student has his or her own work at hand and may glance at it at any time. At the same time he/she can see the board. Accordingly every student has access to two medially graphic elements (see Oesterreicher 1997; Oesterreicher/Koch 1985; Fetzer 2003a) at a time: his/her own written work and the board. Both graphic elements may be either “consistent” or show thematic breaks and be “inconsistent” (Fetzer 2006a).

**Argumentation**

In order to understand the nature of logical processes in publishing situations, I use Toulmin’s approach to argumentation as a tool of analysis (Toulmin 1969; see also Fetzer/Schreiber/Krummheuer 2004). According to this approach, utterances are not
seen from the perspective of what the speaker might have intended to express or what he might have meant. Instead, the aim of the analysis is to reconstruct the function which an individual’s utterance fulfils within the argument. Toulmin’s analysis of argumentation helps to determine those functions.

Based on Toulmin’s work, the shortest possible argument consists of the two elements data and conclusion. The conclusion is the claim that needs to be established when it is challenged; it has to be shown that it is justifiable. The data is our personal knowledge, the facts we appeal to as a foundation for the claim, the ground we produce as support for the original assertion. Such a ‘simple’ argument can be summed up as follows: Data \( D \) is the basis so the conclusion \( C \) can be established. An argumentation gets more complex if warrants are included. Warrants are general, hypothetical statements which can act as bridges and indicate the bearing on the conclusion on the data already produced. You can get from \( D \) to \( C \) since the warrant \( W \). Arguments get even more complex, if backings of warrants are offered. Warrant \( W \) is acceptable in general on account of backing \( B \) (Toulmin 1969, p.94ff.). Toulmin put the structure of arguments into a graphic layout shown in the figure (1).

Most of the time in the mathematics class primary students may get along producing simple arguments consisting only of data and conclusion. But sometimes simple arguments do not meet the requirements of the situation anymore. For example the teacher demands further explanation to the statement \( 3+7=10 \). Warrants might be put forward: I did it by the use of wooden pearls; I counted; Joe figured out the same result. If more complex arguments are expected to fulfil the interactional demands, I describe the interactional situation as “argumentatively condensed” (Fetzer 2006a/b).

**EMPIRICAL RESULTS**

“How does writing improve learning?” (D. Miller 1991, p.516). Now that the terms needed to approach this question have been introduced I am going to describe how interactional situations in publishing phases emerge that enable mathematical learning. To illustrate I start by considering the example introduced above (transcript 2).

**Example**

The task was to lengthen a given line by 6cm 4mm (fig. 2, left). Sonja is the first child to present her approach on the blackboard (fig. 2, right). After Sabina’s question Sonja starts explaining again. Then Benno interferes. Without being asked
he gets hold of the current speaker’s role: “That was eight millimetres.” Regarding the dimension of participation he seizes the turn. How does it happen? What conditions make Benno take over an active role in the interaction and seize the turn? Remember the setting of the publishing phase: Benno has access to the blackboard and his own written work at the same time. Obviously there are inconsistencies between both graphic elements: On the board it says “7”, whereas Benno’s work does not contain the number 7 at all. Instead, his written product shows an eight, but not in the sense of a number but as a measured value: 8mm. Benno recognizes these thematic breaks. He identifies the two graphic elements to be inconsistent and as a consequence he seizes the turn and contradicts: “That was eight millimeters.” At this point the third dimension, the aspect of argumentation, gains relevance. With Benno contradicting he makes the inconsistencies between the graphic elements ‘work’ and ‘board’ explicit to all members of the interaction. He points out the two senses of interpreting the digits, either as a number or as a measured value. Besides he alludes to the different results of measurement (7(mm) and 8mm). With the inconsistencies being explicit and accessible to all members of the interaction, Sonja can’t continue her explanation. There arises the need to negotiate those aspects Benno referred to. Attempting to explain what she did, Sonja started off with a simple argument consisting of data and conclusion: “I have five, that made five up there…” This argumentative level does not meet the requirements of the interaction anymore. The interactional situation condenses argumentatively. Concerning the aspect of learning this is the crucial impetus. In order to take part in the process of interaction

**Figure 2: Task (left), current writing on the board (right)**

**Figure 3: Benno’s work**
current speakers need to put forward more complex arguments. Warrants are required to bridge data and conclusion. Eventually even backings might be demanded. Such argumentatively condensed situations urge the presenting child to re-think her approach (Sonja). Besides, for the student who triggers the argumentatively condensed situation the conditions for learning are beneficial (Benno). Those who are taking a receptive role at the moment are required to contribute to the process of argumentation. Finally, there are those members of the interaction that remain ‘quiet’. They might profit by hearing the complex arguments put forward.

This example shows how the emergence of interactional conditions that enable mathematical learning can be described by the synopsis of the three dimensions participation, thematic development and argumentation. In the given example one student seizes the turn when identifying inconsistencies between both graphic elements. The interactional situation becomes argumentatively condensed.

**Summary**

What interactional conditions enable mathematical learning? How do argumentatively condensed situations emerge? Analyses of numerous episodes of interaction in publishing phases reveal the following empirical results.

Regarding publishing phases there are two conditional settings that contribute to the emergence of argumentatively condensed situations in the mathematics classroom:

- If students identify *inconsistencies* between the own written work and the blackboard and thereupon contribute to the process of interaction actively, argumentatively condensed situations emerge. In this context it does not matter if the students accept, pick up or seize the turn.

- If students contribute to the process of interaction by *seizing the turn*, they evoke the emergence of argumentatively condensed situations.

In both cases students need to create rather complex arguments in order to participate actively in the process of interaction. To fulfil the interactional requirements arguments put forward need to exceed the basic structure of data and conclusion. The interactional conditions of learning optimize.

“How does writing improve learning?” (Miller 1991, p.516). Especially within the *publishing* phase argumentatively condensed situations emerge. Brief designations of the result of a mathematical task as well as the answer “I’ve got the same” are insufficient and do not meet the interactional demands. Instead, explaining and negotiating are appropriate activities to contribute to the process of interaction. Looking at the board one second and glancing at ones own written work the next second can serve as a basis for an attitude of “I did it differently”. The possibility of relating both graphic elements enhances the chance to *identify inconsistencies* between work and board. At the same time the coconstantaneous access to both graphic elements provides self-confidence and thus supports *seizing a turn*. Working
with writing in the mathematics classroom turns the well-known “I’ve got the same” into “I did it differently”, “I did not get it, please explain again”, “How did you proceed?” Argumentatively condensed situations can be regarded as optimised learning conditions for all members of the interaction. Those who think about a coherent and convincing explanation will benefit as well as those who listen to the arguments presented. Students who contribute actively to the emergence of complex arguments will profit as well as children who take a receptive role.

Finally, I focus the title of this article and return to the opening question: “Why should I implement writing in my classes?” (Miller 1991, p. 518).

Based on the results of my empirical study I specify the answer offered in the beginning. Mathematical writing should be implemented in classes because, especially during the publishing phase, chances are good for the emergence of argumentatively condensed situations. If students present their written works, if they explain their proceedings, if they discuss different approaches and have access to their work and the board, optimised learning conditions are likely to emerge.

**TRANSCRIPTION RULES**

The first column indicates the names of the interacting persons. The second and third column give the verbal (regular font) and non-verbal (italic font) actions in English and in German.

<table>
<thead>
<tr>
<th>/ - \</th>
<th>Rising, even, falling pitch.</th>
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<tbody>
<tr>
<td><strong>bold</strong></td>
<td>Accentuated word.</td>
</tr>
<tr>
<td>spaced</td>
<td>Spoken slowly.</td>
</tr>
<tr>
<td>(incomprehensible)</td>
<td>Incomprehensible utterance.</td>
</tr>
<tr>
<td>+</td>
<td>The indicated way of speaking ends at this symbol.</td>
</tr>
<tr>
<td>#</td>
<td>There is no break, the second speaker follows immediately from the first.</td>
</tr>
<tr>
<td>&lt;M four five six\</td>
<td>Indicates where people are talking</td>
</tr>
<tr>
<td>&lt;S five</td>
<td>at the same time.</td>
</tr>
</tbody>
</table>

**REFERENCES**


