How many families survive for a long time

Vatutin V.A.
(Steklov Mathematical Institute, Moscow)

Let $\{Z_k, k = 0, 1, 2, ...\}$ be a critical branching process in random environment and let $Z_{p,n}$ be the number of particles at time $p < n$ in the process having a positive offspring number at time n. We show that if the associated random walk of the branching process belongs to the domain of attraction of a stable law with parameter $\alpha \in (0, 2]$ then there exists a sequence $\left\{c_p^{-1}, p = 1, 2, ...\right\}$ such that the conditional law

$$\mathcal{L}\left(\left\{c_p^{-1} \log Z_{pu}, 0 \le u < \infty\right\} | Z_n > 0\right)$$

weakly converges, as $n \gg p \to \infty$ to the law of an $\alpha-$stable Levy process conditioned to stay nonnegative on the semi-axis $[0, \infty)$.

Basing on this result we prove a conditional functional limit theorem for the properly scaled process $\{\log Z_{pu,n}, 0 \le u < \infty\}$ given $Z_n > 0$ and $n \gg p \to \infty$.

This work is supported by the RSF under a grant 14-50-00005.