Simultaneous imaging of absorption and scattering in dc diffuse optical tomography

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Diffuse optical tomography

Diffuse optical tomography (DOT):

- Transilluminate biological tissue with visible or near-infrared light
- Goal: Spatial image of interior properties.

  Relevant physical quantities (in diffusive regime):
  - Scattering
  - Absorption
- Medical application: Visualize blood-volume and oxygenation for e.g.
  - Breast cancer detection
  - Bedside-imaging of neonatal brain function

Topical reviews on DOT

- Gibson, Hebden and Arridge (2005), Arridge (1999)

Scattering/absorption crosstalk

Most physiological information is in the absorption coefficient.

Can scattering and absorption effects be distinguished?

$\rightsquigarrow$ Mathematical question:

DC diffusion model for photon density $\Phi(\mathbf{r})$ at point $\mathbf{r}$:

$$-\nabla \cdot (\kappa(\mathbf{r}) \nabla \Phi(\mathbf{r})) + \mu_a(\mathbf{r}) \Phi(\mathbf{r}) = 0$$

$\kappa(\mathbf{r})$:	Scattering coefficient
$\mu_a(\mathbf{r})$:	Absorption coefficient
Boundary conditions for $\Phi(\mathbf{r})$:	Inward and outward light flux

Can one reconstruct both $\kappa(\mathbf{r})$ and $\mu_a(\mathbf{r})$ from boundary data of $\Phi(\mathbf{r})$?

Most idealized data model:
Measure bound. values of $\Phi(\mathbf{r})$ and $\nabla\Phi(\mathbf{r})$ for all possible solutions of Φ.

Non-uniqueness

Can one reconstruct both $\kappa(\mathbf{r})$ and $\mu_a(\mathbf{r})$ from boundary data of $\Phi(\mathbf{r})$?

$$-\nabla \cdot (\kappa(\mathbf{r}) \nabla \Phi(\mathbf{r})) + \mu_a(\mathbf{r}) \Phi(\mathbf{r}) = 0.$$

Arridge/Lionheart (1998 Opt. Lett. 23 882–4):

- Transformation $\Psi(\mathbf{r}) := \sqrt{\kappa(\mathbf{r})}\Phi(\mathbf{r})$

$$-\nabla^2 \Psi(\mathbf{r}) + \eta(\mathbf{r}) \Psi(\mathbf{r}) = 0, \quad \eta(\mathbf{r}) := \frac{\nabla^2 \sqrt{\kappa(\mathbf{r})}}{\sqrt{\kappa(\mathbf{r})}} + \frac{\mu_a(\mathbf{r})}{\kappa(\mathbf{r})}.$$

- If $\kappa = 1$ around bound. $\rightsquigarrow$ transformation doesn't affect bound. data.

$\rightsquigarrow$ Boundary data only depends on effective absorption η.

Absorption and scattering effects cannot be distinguished.

(Note: Argument requires smooth scattering coefficient κ).

Experimental results

Theory:

Absorption and scattering effects cannot be distinguished.

Practice:

Pei et al. (2001), Jiang et al. (2002), Schmitz et al. (2002), Xu et al. (2002) successfully reconstructed separate images of absorption and scattering (from phantom experiment using dc diffusion model!)

$\rightsquigarrow$ Practice contradicts theory!

Pei et al. (2001):

"As a matter of established methodological principle (...) empirical facts have the right-of-way; if a theoretical derivation yields a conclusion that is at odds with experimental results, the reconciliatory burden falls on the theorist, not on the experimentalist."

New uniqueness result

Common situation in practice:

- Composite tissue, made up of regions with different scattering and absorption properties.
- Across region boundaries: jumps in coefficients and/or derivatives.

$\rightsquigarrow$ Non-uniqueness result does not hold for these cases.

Theorem *(H., Inverse Problems 25, 055010 (14pp), 2009)*

For composite tissue with

- piecewise constant scattering
- piecewise smooth (real-analytic) absorption

measurements of dc DOT determine both scattering and absorption.

New uniqueness result

Theorem *(H., Inverse Problems 25, 055010 (14pp), 2009)*

For composite tissue with

- piecewise constant scattering
- piecewise smooth (real-analytic) absorption

measurements of dc-DOT determine both scattering and absorption.

- Piecewise constantness seems fulfilled for phantom experiments.

$\rightsquigarrow$ Result reconciles theory with practice.

- Measurements contain more than just the effective absorption!

Precise characterization

- Arridge/Lionheart (1998): No uniqueness for general smooth (κ, μ_a).
- H. (2009): Uniqueness for piecewise constant κ, piecew. smooth μ_a.

What information about (κ, μ_a) do the measurements contain?

Precise characterization *(H., rigorous proof submitted for publication)*

Measurements precisely determine

- η where κ and μ_a are smooth,
- (weighted) jumps in κ and $\nabla\kappa$.

Intuitive (non-mathematical!) reason:

$$\eta = \frac{\nabla^2 \sqrt{\kappa}}{\sqrt{\kappa}} + \frac{\mu_a}{\kappa}.$$

Jumps in κ or $\nabla\kappa$ $\quad\rightsquigarrow\quad$ distributional singularities in $\Delta\sqrt{\kappa}$.

Conclusions

DC intensity measurements in diffuse optical tomography

- cannot distinguish smooth scattering effects from absorption effects.
- but they do determine
  - a combination of scattering and absorption coefficient (the ”effective absorption“),
  - jumps in the scattering coefficient and its derivative.

(Theoretically,) dc DOT can simultaneously show

- absorption and scattering if the tissue has piecew. linear scattering ($\rightsquigarrow$ reconciles theory with experimental results).
- the effective absorption and the jumps in the scattering (and its derivative) for general composite materials.