Simultaneous imaging of absorption and scattering in dc diffuse optical tomography

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Abstract— **We present new results on the fundamental non-uniqueness issue in dc diffuse optical tomography (DOT) that resolve a long-standing conflict between theory and practice. Theoretically, scattering and absorption properties of the imaged media were proven to produce equivalent and thus indistinguishable effects. However, successful simultaneous reconstructions of both parameters were obtained in phantom experiments.**

We reconcile theory and practice by showing that the equivalence of scattering and absorption is only true for spatially smoothly varying scattering properties. Jumps in the scattering properties are however distinguishable from absorption effects. In fact, in composite tissue consisting of regions with constant scattering properties, dc intensity measurements in DOT are sufficient to simultaneously image both the absorption and the scattering coefficient.

We give an intuitive justification for this result and conjecture a complete theoretical characterization of what can be reconstructed from dc intensity measurements in DOT. Essentially, the reconstructable quantities consist of the effective absorption properties together with the jumps of the scattering coefficient and its first derivative.

Keywords— **diffuse optical tomography (DOT), dc intensity measurements, unique simultaneous recovery of scattering and absorption, inter-parameter crosstalk**

I. Introduction

Diffuse Optical Tomography (DOT) aims to reconstruct spatially resolved images of the scattering and absorption properties of biological tissue by transilluminating it with light in the near-infrared regime, cf. Arridge [1] and Gibson *et al* [2] for topical reviews. A long-standing conflict between theory and practice is the question whether dc intensity measurements hold sufficient information to simultaneously image both properties.

A. Non-uniqueness in diffuse optical tomography

Rigorous theoretical results of Arridge and Lionheart [3] demonstrate that this goal cannot be accomplished with dc intensity measurements alone. The reason is a fundamental non-uniqueness property of the underlying mathematical model in which scattering and absorption properties are convertible into each other. It is therefore theoretically impossible to determine whether an experimentally observed effect is due to absorption or scattering.

Due to this result, every reconstruction algorithm is faced with an infinite number of absorption/scattering combinations that all comply with the measurements. Some of these combinations may be sorted out by practical arguments as they may have too large oscillations or lie without expected realistic bounds. Also, some may lie sufficiently close to the real values, so that one would consider them as acceptable approximations.

Nevertheless, the arguments from [3] still provide an arbitrary number of physically reasonable and significantly distinct combinations. Hence, no reconstruction algorithm (that relies on the dc diffusion model) should be able to pick to right one.

Experimental results are, however, in striking contrast to these theoretical findings. Pei *et al* [4], Jiang *et al* [5], Schmitz *et al* [6] and Xu *et al* [7] successfully reconstructed separate images of the absorption and scattering parameters using only dc measurements. Notably, they used the dc diffusion model for which theoretical non-uniqueness was proven.

No satisfactory explanation for this apparent gap between theory and practice has been found so far. Are the practical successes really just due to the more or less implicit use of a-priori information contained in a sophisticate numerical implementation and regularization?

B. New uniqueness results

In this work we describe a recent theoretical result [8] on this fundamental non-uniqueness issue that reconciles the theory with the experimental results and leads to important practical implications. Our aim is to explain the new ideas in a intuitive rather than rigorous manner to appeal to more practically oriented readers. Based on these intuitive arguments we also formulate a conjecture containing the complete theoretical characterization of what can be reconstructed from dc intensity measurements in DOT.

Our main new point is that previous theoretical arguments relied on the assumption that the parameters are everywhere

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spatially smooth. In practice, however, DOT is used to image composite tissue, containing regions with different physical properties. Moreover, finding the region boundaries is often one of the major goals in DOT.

In fact, our new theoretical results show that such region boundaries can be identified from boundary measurements in dc diffuse optical imaging. More precisely, [8] shows that for composite tissue consisting of regions with constant scattering properties, dc intensity measurements in DOT are sufficient to simultaneously image both the absorption and the scattering coefficient. This explains the successful simultaneous reconstructions in the practical experiments cited above, where phantoms with approximately piecewise constant properties where used.

In addition to this rigorously proven result we derive a conjecture that completely characterizes what can be reconstructed in the case of general composite tissue. Essentially, the reconstructable quantities consist of the effective absorption properties together with the jumps of the scattering coefficient and its first derivative.

The outline of this work is as follows. In the next section we recall the non-uniqueness result of [3] and demonstrate how smooth scattering and absorption coefficients can be combined in one effective absorption parameter. This is followed by a formal derivation of the effective absorption for a medium that is composed of several different materials which motivates our results on what can be identified on the regions boundaries. We then state our theoretical uniqueness results in a more precise form. In the final section we give some conclusions.

II. Theoretical crosstalk between scattering and absorption

We recall the theoretical arguments of [3] why smooth scattering and absorption properties are convertible into each other and thus indistinguishable by dc intensity measurements alone.

A. *The mathematical model*

The equation governing dc diffuse optical tomography is

$$-\nabla \cdot (\kappa(\mathbf{r})\nabla\Phi(\mathbf{r})) + \mu_a(\mathbf{r})\Phi(\mathbf{r}) = 0 \qquad (1)$$

in some domain Ω, where $\Phi(\mathbf{r})$ is the intensity distribution at the point $\mathbf{r} \in \Omega$, $\kappa(\mathbf{r})$ is the scattering, and $\mu_a(\mathbf{r})$ is the absorption coefficient. This equation is supplemented by boundary conditions describing the applied inward and the measured outward light flux. For the purpose of this work, we assume that the measurements provide information of the intensity on the boundary $\Phi(\mathbf{r})|_{\partial\Omega}$ (the Dirichlet data) and its normal derivative $\partial_\nu\Phi(\mathbf{r})|_{\partial\Omega}$ (the Neumann data) for all solutions of the diffusion equation (1).

Note that there exists more sophisticated and more realistic ways of modeling the boundary conditions. These include the use of delta sources located at a small depth below the surface of Ω, or Robin boundary conditions possibly including the effects of boundary reflections. Nevertheless, our simplified boundary model may serve as a kind of ”best you can get”, idealized data for the study of theoretical uniqueness. The reason is that Dirichlet and Neumann data, $\Phi(\mathbf{r})|_{\partial\Omega}$ and $\partial_\nu\Phi(\mathbf{r})|_{\partial\Omega}$, on a closed surface completely determine $\Phi(\mathbf{r})$ inside the surface. Thus, if we replace a more sophisticated boundary model on $\partial\Omega$ by a full set of Neumann and Dirichlet data on a slightly smaller surface, we are only increasing but never decreasing the theoretically available information in the data. cf. also [3] on this argument.

B. *Combining absorption and scattering effects*

The transformation $\Psi(\mathbf{r}) = \sqrt{\kappa(\mathbf{r})}\Phi(\mathbf{r})$ simplifies the diffusion equation (1) into

$$-\nabla^2\Psi(\mathbf{r}) + \eta(\mathbf{r})\Psi(\mathbf{r}) = 0,$$

with

$$\eta(\mathbf{r}) := \frac{\nabla^2\sqrt{\kappa(\mathbf{r})}}{\sqrt{\kappa(\mathbf{r})}} + \frac{\mu_a(\mathbf{r})}{\kappa(\mathbf{r})}$$

Thus, the scattering coefficient κ and the absorption μ_a are combined into one effective absorption parameter η. If κ is identical to one, close to the outer boundary $\partial\Omega$, then the measured Neumann and Dirichlet data remains unchanged by this transformation. In effect, the measurements do only depend on the effective absorption coefficient which combines scattering and absorption. In other words, for spatially smoothly varying coefficients, it is impossible to determine whether a certain measured effect is due to the tissues scattering or absorption properties.

III. Effective absorption of composite tissue

In practice, the tissue will be composed of regions with different optical properties. Along the region boundaries the scattering coefficient κ may be discontinuous, so that the above combination of both, scattering and absorption, into one effective absorption parameter is no longer valid.

We now nevertheless study the effective absorption

$$\eta(\mathbf{r}) = \frac{\nabla^2 \sqrt{\kappa(\mathbf{r})}}{\sqrt{\kappa(\mathbf{r})}} + \frac{\mu_a(\mathbf{r})}{\kappa(\mathbf{r})} \tag{2}$$

for composite tissue. Note that this expression does not make sense from a rigorous mathematical point of view. However, we believe that the formal arguments in this section provide a useful motivation for our new (rigorously proven) uniqueness results.

Consider first the one-dimensional example on the interval $x \in (-1,1)$ with

$$\kappa(x) := \begin{cases} 1 & \text{for } -1 < x \leq 0, \\ 2+x & \text{for } 0 < x < 1, \end{cases} \qquad \mu_a(x) := 1.$$

In one dimension, the Laplacian ∇^2 is the second derivative. Since $\kappa(x)$ and also its derivative $\kappa'(x)$ have jumps in the point $x = 0$, the derivatives of κ contain singularities in the form of the δ-distribution. More precisely,

$$\kappa'(x) = \delta(x) + \Theta(x),$$

where

$$\Theta(x) := \begin{cases} 0 & \text{for } x \leq 0, \\ 1 & \text{for } x > 0 \end{cases}$$

is the Heaviside step function. Hence,

$$\nabla^2 \kappa(x) = \kappa''(x) = \delta'(x) + \delta(x).$$

Similarly, in two or three dimensions, a jump of the scattering coefficient or its normal derivative along some region boundary will lead to distributional singularities supported on this boundary. Hence, the first summand in the effective absorption (2) contains singularities that cannot be canceled out by the non-singular second summand in (2).

This suggest that, for composite tissue, dc optical measurements do not only contain information about the effective absorption on each part of the tissue, but also about the region boundaries and the jumps of the scattering parameter (and its first derivative) along these boundaries. In other words, smooth deviations in the scattering properties may not be distinguishable from deviations in the absorption, but jumps in the scattering parameter (and its derivative) can be determined!

IV. The new uniqueness results

Now we state our new uniqueness results. We start with the result published in [8].

Theorem 1
Let the medium be composed of regions in which the scattering is constant and the absorption is smooth (real-analytic). Then the measurements of dc diffuse optical tomography uniquely determine both κ and μ_a. □

From our formal motivation above we may justify Theorem 1 by the following arguments. The measurement data contains information about the effective absorption η on each tissue part and also on the jumps of κ. A priori knowing that κ is piecewise constant, enables us to calculate κ by summing up all jumps necessary to reach a certain tissue part (starting from the "first" jump on the outer boundary). Then (since $\nabla^2 \kappa = 0$) we can calculate μ_a from η.

Theorem 1 already solves the apparent conflict between theoretical and practical results described in the introduction. In phantom experiments, the medium is usually composed of rather simple parts, so that the assumption of piecewise constant scattering properties seems well met. Theorem 1 thus reconciles the successful simultaneous reconstructions of both parameters in the practical experiments with the theory.

The following assertion from [9] precisely characterizes what can be reconstructed in dc diffuse optical tomography for composite tissue. At the point of writing this work, the rigorous mathematical proof of this assertion has not been verified independently yet. Hence, we formulate the characterization as a conjecture.

Conjecture 2
Let the medium be composed of regions in which both scattering and absorption are smooth (real-analytic). Then the measurements of dc optical tomography uniquely determine

- the scattering κ and its normal derivative $\partial_\nu \kappa$ on the outer boundary,
- the effective absorption η on all regions,
- the relative jumps of the absorption κ on the region boundaries, i.e. the quotient κ^+/κ^- of its one-sided values κ^+ and κ^-.
- the jump of $\partial_\nu \kappa$ weighted with a one-sided value on the region boundaries, i.e.,

$$\frac{\partial_\nu \kappa^+ - \partial_\nu \kappa^-}{\kappa^+},$$

cf. Figure 1 for the one-sided values κ^+ and κ^-.

Conversely, two different tissues can only be distinguished by dc intensity measurements if their properties differ in at least one of the above points. □

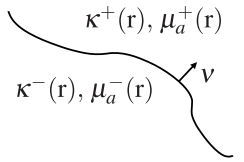

Fig. 1: Region boundary in composite tissue.

V. Conclusion

We have presented theoretical uniqueness results in dc diffuse optical tomography and reconciled the theory with the experimental results. Dc intensity measurements do not hold enough information to distinguish smooth scattering effects from smooth absorption effects. They do, however, suffice to uniquely identify jumps in the scattering (and its derivative).

For practical reconstructions, Theorem 1 shows that unique simultaneous reconstruction of both scattering and absorption coefficients is possible if the scattering is a priori known to be piecewise constant. Conjecture 2 asserts that this is still the case for piecewise linear (or piecewise harmonic) scattering. If such priors are not justifiable, then the conjecture yields that one can still image the effective absorption together with the region boundaries and jumps of the scattering parameter (and its derivative).

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References

1. Arridge S R (1999) Optical tomography in medical imaging. Inverse Problems 15:R41–R93 DOI 10.1088/0266-5611/15/2/022
2. Gibson A P, Hebden J C, Arridge S R (2005) Recent advances in diffuse optical imaging. Phys. Med. Biol. 50:R1–R43 DOI 10.1088/0031-9155/50/4/R01
3. Arridge S R, Lionheart W R B (1998) Nonuniqueness in diffusion-based optical tomography. Opt. Lett. 23:882–884 DOI 10.1364/OL.23.000882
4. Pei Y, Graber H L, Barbour R L (2001) Normalized-constraint algorithm for minimizing inter-parameter crosstalk in dc optical tomography. Optics Express 9:97–109.
5. Jiang H, Iftimia N V, Xu Y, Eggert J A, Fajardo L L, Klove K L (2002) Near-infrared optical imaging of the breast with model-based reconstruction. Acad. Radiol. 9:186–94.
6. Schmitz C H, Locker M, Lasker J M, Hielscher A H and Barbour R L (2002) Instrumentation for fast functional optical tomography. Rev. Sci. Instrum. 73: 429–39 DOI 10.1063/1.1427768
7. Xu Y, Gu X, Khan T, Jiang H (2002) Absorption and scattering images of heterogeneous scattering media can be simultaneously reconstructed by use of dc data. Applied Optics 41:5427–5437 DOI 10.1364/AO.41.005427
8. Harrach B (2009) On uniqueness in diffuse optical tomography. Inverse Problems 25:055010(14pp) DOI 10.1088/0266-5611/25/5/055010
9. Harrach B (2009) Simultaneous determination of the diffusion and absorption coefficient from boundary data. Submitted for publication.

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